

Mathematics Foundation Course B: Modeling Our World with Mathematics

Quest University

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Mathematics underlies almost every human activity in the modern world. It permeates our understandings of the physical world, populations (human and ecological), time and space, and networks, to name just a few. It affects our actions in social choice, the shaping of public opinion, and consumer and business decisions. The process that turns reality into mathematics and back — modeling — is the backbone of our quantitative understanding of our world.

We begin with an exploration of the use of functions to model continuous change: linear and exponential functions for food production and population growth, trigonometric functions for periodic events, statistical distributions for behaviours of large groups, and so forth. We then move on to discrete rather than continuous phenomena: for instance, graph theory for networking, coding and cryptography, elections and voting paradoxes, game theory, and the prisoner's dilemma.

TEXTS:

Required: Photocopied package, containing sections from Warren Page's draft textbook. Available only in the bookstore.

Recommended:

- Eric Connally et al., Functions Modeling Change, 2nd edition. New York: Wiley, 2003.
- Peter Tannenbaum, Excursions in Modern Mathematics, 6th edition. Upper Saddle River, NJ: Prentice Hall, 2007.

Both recommended books are available on the Internet in older editions for cheap. You do not need the most current edition!

Both books will be available in the Learning Commons, and on reserve in the library. I recommend purchase if like to have them always within reach and tend to rely on textbooks.

SOFTWARE:

Curvus Pro for the Macintosh (now incorporated in Mac OS X 10.4 and later as *Grapher*), or *Graphmatica* for Windows (www.graphmatica.com); both are available on the network. Both are shareware and can be downloaded onto your own machines; you can register simply by recording your name with me.

DAILY PATTERN:

- 9 AM: Discussion of issues arising from previous day's work
 - 9:15 AM: Introduction of new topic, beginning with the class together and doing regular group activities
 - 10:20 AM – 10:30 AM: Summary of results, practice activity (not evaluative) distributed, to be completed before the afternoon class

 - 1 PM: Regroup to discuss the practice activity
 - 1:15 PM: Develop the topic in more depth — in-class discussion, further activities, or problem solving in small groups, as the situation dictates
 - 2:20 PM – 2:30 PM: Introduction of daily problem set
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ASSESSMENT:

☐ Daily short problem sets	55%
☐ Midterm synthesis assignment	15%
☐ Final synthesis assignment	15%
☐ Participation / initiative	15%

Problem Sets: Every afternoon at 2:30 you will receive a short assignment based on the material discussed that day. You are always free to use *Curvus Pro* or *Graphmatica* (or whatever else you have chosen to use). Feel free to discuss these problems with each other, but please make sure that you hand in your own work, not something dictated from someone else. I hope never to receive two identical assignments with different names on them.

Grading: I will evaluate about half of the work you turn in. If there is something you particularly wish me to look at, please indicate this at the top of the page in ***bright red ink***.

Midterm/Final Synthesis Assignments: At the middle and end of the block there will be an evaluation of your understanding, in the form of an assignment that synthesizes the material learned in the previous ten days. Although the format will be similar to a regular assignment, they will be done strictly individually; no discussing or consulting.

Learning Outcomes Grid

<i>Learning Outcome</i>	<i>Class activities</i>	<i>Assessment</i>
College-wide		
Critical thinking	Daily problem-solving exercises	Daily problem sets; synthesis assignments
Communication	Sharing of group results with the class	Daily problem sets; synthesis assignments; participation
Integration	All problems arise from various empirical contexts	Daily problem sets; synthesis assignments
Math foundation		
Understanding abstraction	All in-class problems begin with abstraction from context	Daily problem sets; synthesis assignments
Recognizing types of abstraction	Converting between visual, numeric, symbolic representations is major theme	Daily problem sets; synthesis assignments
Choosing appropriate models	This is the topic of the course	Daily problem sets; synthesis assignments
Selecting problem-solving strategies	First half: functional models; second half: more general models	Daily problem sets; synthesis assignments
Applying/evaluating tools	Excel, graphing software used throughout	Daily problem sets; synthesis assignments
Placing results in context	All in-class problems conclude with this	Daily problem sets; synthesis assignments

Course Outline and TENTATIVE Schedule

<i>Date</i>	<i>Topic</i>	<i>Section in Page draft text</i>	<i>Section in Connally (C) or Tannenbaum (T)</i>
Day 1	Introductions; the modeling process; linear functions and graphs	3.1	C 2.1, 2.2
Day 2	Linear functions as approximations: regression and tangent lines	3.2-3.3	C 2.4
Day 3	Polynomials; polynomial and power regression	3.4-3.5	C 9.1, 9.3
Day 4	Population and other forms of growth: exponential functions	4.1	C 4.1
Day 5	Logistic growth; inverting the exponential relation	2.4	C 4.2
Day 6	Approximating growth: exponential regression	4.3	
Day 7	Modeling data: using statistical distributions		
Day 8	Trigonometry: origins and modeling periodic functions	5.1, 5.2	C 6.1 – 6.5
Day 9	Midterm synthesis assignment		
Day 10	Graph theory, routing problems, and networks		T5
Day 11	The traveling salesman problem; trees		T6, T7
Day 12, 13	The mathematics of voting: paradoxes of democracy		T1
Day 14	Fair division; apportionment problems in political bodies		T3, T4
Day 15	Game theory; the prisoner's dilemma		<i>For All Practical Purposes 15</i>
Day 16	Scheduling problems		T8
Day 17	Spiral growth; fractal geometry		T9, T12
Dec. 18	Final synthesis assignment		

The second half of the schedule is especially tentative. Other topics we could explore include the use and abuse of statistics, symmetry, probability theory, cryptography and coding systems, tilings, and non-Euclidean geometries. ***You will help choose the topics!***

Sample Group Activities

ACTIVITY: The Rolling Ball

Form groups of *four or five* students for this activity.

Items needed:

- A ball (tennis, soccer, ping pong, whatever)
- A meter stick
- A digital watch, preferably but not necessarily with a stop watch

To get a sense of the power and difficulty of modeling with Cartesian coordinates, we will try to model the path of a rolling ball.

1. Find a good clear path. One person will launch the ball. One person counts off the seconds. One person determines where the ball is after one second, and two seconds. Another person records the spots where the ball is after three and four seconds. If you have a fifth person, that person can do five and six seconds.

A WORD TO THE WISE: Try to roll the ball so that it travels a good length, but stops before it reaches the end of the room!

2. Make a numerical chart of the total distance traveled for each second.
3. Now draw Cartesian axes where time (called t) is measured on the horizontal axis, and distance (called d) is measured vertically. Plot your points on the graph.
4. Your points should trace out a smooth curve. If they don't, explain why something must have gone wrong, and (*yes, you guessed it!*) go back to Step 1.
5. **WITHOUT LOOKING AT ANYTHING BUT YOUR GRAPH**, make a numerical estimate of how far the ball would have traveled altogether.
6. How do you know that the curve cannot be a line, or a circle, or even a parabola? Explain carefully.

CHALLENGE: Draw the best line you can through your graph, **and** the best circle that you can. Think of a good way to evaluate which of these two curves is a better model for your data. Explain clearly your criteria.

ACTIVITY: The Bay of Fundy: The Tides of Change

Form groups of *three* students for this activity.

Items needed:

- Graphmatica or Curvus Pro
- A web browser

The Bay of Fundy in Nova Scotia has the most spectacular tides in the world. Between low and high tides, the difference in height is 15 meters --- nearly 50 feet! Suppose that a ruler is constructed alongside a pier, from the sea base to the top of the pier. The water's height varies between 1.4 meters and 16.4 meters. For those of you who are not seafarers, ahoy matey! The difference in time between successive high tides is 12.4 hours.

Recall the basic sine model: $y = a + b\sin(cx + d)$

1. Let x be time measured in hours, and let y be the water's height. Given the information above, you should be able to determine *at least* two of the four parameters in our model, without using any data. Which ones, and what are their values? Explain your reasoning.
2. It is actually possible to determine a *third* parameter. Likely the one you missed is c . What piece of information do you need, from above, to calculate c ? See if you can find its value. (This is the CHALLENGE problem...)
3. This leaves only one parameter to determine, and there isn't enough information above to find its value. Suppose you go out to the dock at midnight (when $t = 0$), and note that the water level is 6.2 meters.

Use the least-squares nonlinear curve fitter with this one data point to determine the value of the last parameter, and use the equation you have now obtained to predict what the water height will be when you begin your fishing day at 4 AM.

Plot your function. WHY IS THERE A PRETTY GOOD CHANCE THAT YOUR PREDICTION WILL BE WAY OFF?

From the above, give a clear one-sentence explanation of a substantial problem with using the curve fitter in this situation.

ACTIVITY: Stellar Observations

Form groups of *three* students for this activity.

Items needed:

- Wooden quadrants and string (supplied)
- A protractor
- A ruler

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1. Choose some object in the room that is above desk level (since we don't have any stars, other things will have to do!). One group member should hold the quadrant vertically while another person looks up at the object from behind the circle. Hold a string so that it points up to the object, and passes through the center of the quadrant.
 2. Lie the quadrant flat; the third person should make sure that the string's orientation remains unchanged. Measure the angle of elevation of the string using a protractor.
 3. Use your ruler to measure the height of the object above the "ground" (the horizontal line through the middle of the quadrant). Use millimeters to retain accuracy. Also, measure the distance to the right of the vertical line through the middle of the quadrant.
 4. Convert your measurements to units in terms of the radius of the circle. (This will require measuring the radius!)
 5. REPEAT STEPS 1-4 FOR SEVERAL OBJECTS, AS TIME PERMITS. WE WILL GATHER OUR MEASUREMENTS TOGETHER.

CHALLENGE: If you have time, try to draw a graph of the height of the object as a function of the angle. Do the same for the rightward distance. Can you make some inferences about the two graphs?

ACTIVITY: The Prisoner's Dilemma

Form groups of *three* students.

Items needed:

- Ozzie
- Harriet
- One person to record the game results
- Two different color pens, if possible

1. Choose players for the three roles outlined above. Players will keep these roles for the entire activity. Now play the game several times in a row, to get a feel for it. ***Remember that Ozzie and Harriet may NOT communicate!***
2. Together, construct a game table for the Prisoner's Dilemma. Just as in the Coke/Pepsi example, put enough space in each box for two entries (one representing Ozzie's score, the other Harriet's).

Since it's hard to quantify the values of the various sentences, we'll use a ranking. Let 4 = the best possible outcome, 3 = the next best, 2 = next best after that, and 1 = worst. First enter the numbers in the game table from Ozzie's point of view; then (with a different pen color) do the same for Harriet.

3. Play the game ten times. The recorder should keep track of the results for both Ozzie and Harriet separately. Both players should try to achieve the highest average score over the ten games. (You can repeat or extend this question if you wish!)
4. Now that you have a feel for the game, come together and consider carefully the game from Ozzie's point of view only. Explain why it is in Ozzie's best interest to confess, regardless of what Harriet does. Then do the same, for Harriet's point of view.
5. If both players take the reasoning of Q4, what is the inevitable result of the Prisoner's Dilemma? How would their average scores compare with the average scores Ozzie and Harriet received in Q3?